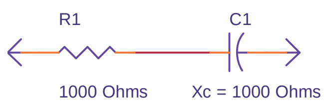
ECT150
Homework #10 Problem Set Handout - RL, RC Phasor Analysis
Sr. Professor Wheeler

Total Points: 24 (3 per problem)
All work must be shown, and final answers boxed or underlined. No credit if work is not shown.

1. Calculate the total impedance ($Z_T$) for the following circuits. Express your answers in both rectangular and polar form. Characterize each impedance as either capacitive, inductive, or purely resistive. (State why).

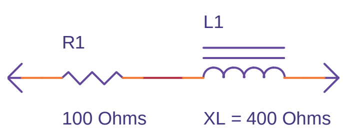
a)




$$Z_T = R - jXc = \underline{\underline{1000 - j1000\Omega}} = \sqrt{1000^2 + 1000^2}\angle\tan^{-1}\left(\frac{-1000}{1000}\right) = \underline{\underline{1.41k\Omega\angle -45°}}$$

Impedance is capacitive due to the negative angle caused by Xc.

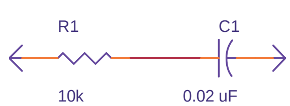
b)




$$Z_T = R + jXL = \underline{\underline{1000 + j400\Omega}} = \sqrt{1000^2 + 400^2}\angle\tan^{-1}\left(\frac{400}{1000}\right) = \underline{\underline{1077\Omega\angle 21.8°}}$$

Impedance is inductive due to the positive angle caused by XL.

2. Calculate the total impedance of the circuit below at the following frequencies: 1000 Hz; 10 kHz. What happens to $Z_T$ as frequency increases, and why?




a) At 1000 Hz:

$$Xc = \frac{1}{2\pi fC} = \frac{1}{2\pi(1kHz)(0.02uF)} = 7958\Omega$$

$$Z_T = R - jXc = \underline{\underline{10k - j7958\Omega}} = \sqrt{10k^2 + 7958^2}\angle\tan^{-1}\left(\frac{-7958}{10000}\right) = \underline{\underline{12.78k\Omega\angle -38.5°}}$$

*(Problem 2 continued on next page)*

*(Problem 2, continued):*

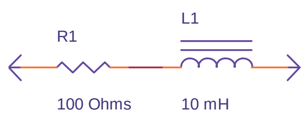
b) At 10 kHz:

$$Xc = \frac{1}{2\pi fC} = \frac{1}{2\pi(10kHz)(0.02uF)} = 795.8\Omega$$

$$Z_T = R - jXc = \underline{\underline{10k - j795.8\Omega}} = \sqrt{10k^2 + 795.8^2}\angle\tan^{-1}\left(\frac{-795.8}{10000}\right) = \underline{\underline{10.03k\Omega\angle -4.6°}}$$

c) As $f$ increases, $Z_T$ decreases, approaching 10 kΩ asymptotically. At 10 kHz, the capacitive reactance Xc is small compared to the series resistance. Therefore, at 10 kHz, the total impedance looks nearly resistive because the capacitor is a short to high frequencies.

3. Calculate the total impedance of the circuit below at the following frequencies: 1000 Hz; 10000 Hz. What happens to $Z_T$ as frequency increases, and why?




a) At 1 kHz:

$$XL = 2\pi fL = 2\pi(1kHz)(10mH) = 62.83\Omega$$

$$Z_T = R + jXL = \underline{\underline{100 + j62.83\Omega}} = \sqrt{100^2 + 62.83^2}\angle\tan^{-1}\left(\frac{62.83}{100}\right) = \underline{\underline{118\Omega\angle 32.1°}}$$

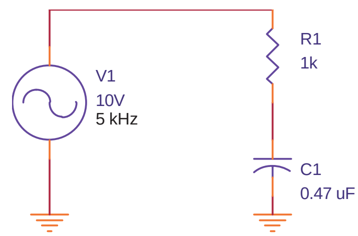
b) At 10 kHz:

$$XL = 2\pi fL = 2\pi(10kHz)(10mH) = 628.3\Omega$$

$$Z_T = R + jXL = \underline{\underline{100 + j628.3\Omega}} = \sqrt{100^2 + 628.3^2}\angle\tan^{-1}\left(\frac{628.3}{100}\right) = \underline{\underline{636.2\Omega\angle 80.96°}}$$

c) As $f$ increases, $Z_T$ increases due to the increase in $X_L$. At a very high frequency, $Z_T$ would be very high since inductors block very high frequencies. Near DC, the impedance would approach 100 Ω since that is the value of the series resistor, and inductors are shorts to DC.

4. In the circuit below, solve for $I_T$ and $V_{R1}$ using phasor methods. Express in both polar and rectangular form.




$$Xc = \frac{1}{2\pi fC} = \frac{1}{2\pi(5kHz)(0.47uF)} = 67.73\Omega$$

$$Z_T = R - jXc = 1k - j67.73\Omega = \sqrt{1k^2 + 67.73^2}\angle\tan^{-1}\left(\frac{-67.73}{1000}\right) = 1002\Omega\angle -3.87°$$

$$I_T = \frac{V_T}{Z_T} = \frac{10V\angle 0°}{1002\Omega\angle -3.87°} = \underline{\underline{9.98mA\angle 3.87°}} = \underline{\underline{(0.00995 + j0.00067)mA}}$$

$$V_{R1} = I_T R_1 = (9.98mA\angle 3.87°)(1000\Omega\angle 0°) = \underline{\underline{9.98V\angle 3.87°}} = \underline{\underline{(9.95 + j0.674)V}}$$

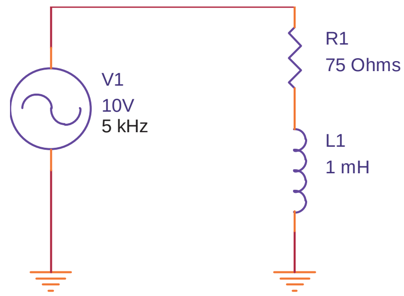
5. In the circuit of problem #4, solve for $V_{C1}$. Demonstrate KVL by adding $V_{C1}$ and $V_{R1}$ (you should get the source voltage).

$$V_{C1} = I_{C1}Z_{C1} = (9.98mA\angle 3.87°)(67.73\Omega\angle -90°) = \underline{\underline{0.676V\angle -86.1°}} = (0.0457 - j0.674)V$$

KVL states that the sum of the rises equals the sum of the drops. By adding $V_{C1}$ and $V_{R1}$ we can verify this:

$$V_T = V_{C1} + V_{R1} = (0.0457 - j0.674)V + (9.95 + j0.674)V = \underline{\underline{(10 + 0j)V = 10V\angle 0° = V_T}}$$

6. In the circuit below, solve for $V_{R1}$ and express in polar form.



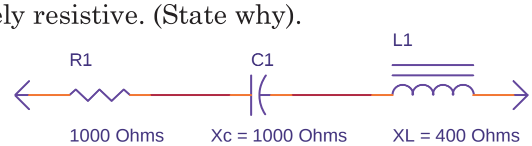

$$XL = 2\pi fL = 2\pi(5kHz)(1mH) = 31.42\Omega$$

$$Z_T = R + jXL = \underline{\underline{75 + j31.42\Omega}} = \sqrt{75^2 + 31.42^2}\angle\tan^{-1}\left(\frac{31.42}{75}\right) = \underline{\underline{81.32\Omega\angle 22.73°}}$$

$$I_T = \frac{V_T}{Z_T} = \frac{10V\angle 0°}{81.32\Omega\angle 22.73°} = 123mA\angle -22.73°$$

$$V_{R1} = I_{R1}R_1 = (123mA\angle -22.73°)(75\Omega) = \underline{\underline{9.22V\angle -22.73°}}$$

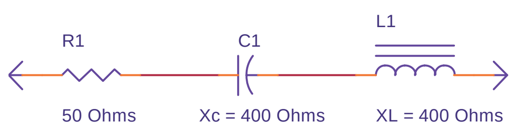
7. What is the total impedance of the circuit below? Characterize it as either capacitive, inductive, or purely resistive. (State why).




a) $Z_T = R - jXc + jXL = (1000 - j1000 + j400)\Omega = \underline{\underline{(1000 - j600)\Omega}}$

b) The circuit is <u>capacitive</u> due to the predominance of Xc (-j600 Ohms).

8. What is the total impedance of the circuit below? Characterize it as either capacitive, inductive, or purely resistive. (State why).




a) $Z_T = R - jXc + jXL = (50 - j400 + j400)\Omega = \underline{\underline{50\Omega}}$

b) The circuit is <u>purely resistive</u>. There is no Xc or XL.