ECT150
Homework #8 Problem Set
Sr. Professor Wheeler

Chapter 17 problems 1-7, 9
Chapter 13 problems 1-7

Total Points: 30 (2 per problem)

All work must be shown, and final answers boxed or underlined. No credit if work is not shown.

(Chapter 17)

1. The capacitor plate area doubles while the dielectric thickness is halved. What is the relationship of the new capacitance value to the original? The dielectric material has not changed.

   Since $C = \frac{\varepsilon A}{d}$ we can determine the effect of each of the two changes:

   Doubling *area* (A) causes a 2X increase in capacitance. Halving *distance* (d), the dielectric thickness, also causes a 2X increase. The combined effect is therefore to increase the capacitance by a factor of four (4).

2. What charge is a 100 pF capacitor storing if it holds 100 V?

$$Q = CV = (100\,pF)(100V) = \underline{\underline{1\times 10^{-8}\,C = 10nC}}$$

3. What capacitance stores 200 pC when charged to 50V?

$$C = \frac{Q}{V} = \frac{200\,pC}{50V} = \underline{\underline{4\,pF}}$$

4. What voltage is present across a 10 µF capacitor having a charge of 2,000 µC?

$$V = \frac{Q}{C} = \frac{2000\mu C}{10\mu F} = \underline{\underline{200V}}$$

5. What is the total capacitance of a circuit containing 100 pF, 400 pF, and 1000 pF capacitors in series? Parallel?

a) Series capacitors act like parallel resistors. The total series capacitance is:

$$C_T = \frac{1}{1/C1 + 1/C2 + 1/C3} = \frac{1}{1/100pF + 1/400pF + 1/1000pF} = \underline{\underline{74pF}}$$

TIP: The total capacitance of series capacitors is always smaller than the smallest capacitance of set.

b) Parallel capacitors add like series resistors:

$$C_T = C1 + C2 + C3 = 100pF + 400pF + 1000pF = \underline{\underline{1500pF}}$$

6. Three capacitors valued at 10 µF, 20 µF, and 50 µF are connected in series across a 160V source. What voltages appear across each capacitor? What value of charge is on each capacitor?

a) SERIES CAPACITORS ALWAYS CARRY THE SAME CHARGE (Q). Therefore:

$Q = C_T V_T$ expresses the charge held on each capacitor.

$$C_T = \frac{1}{1/10\mu F + 1/20\mu F + 1/50\mu F} = 5.88\mu F$$

$$Q = CV = (5.88\mu F)(160V) = \underline{\underline{941\mu C}}$$

<u>Each of the three capacitors stores 941 µC of charge</u>.

b) The voltage across each capacitor can be found by:

$$V_{C1} = \frac{Q_{C1}}{C1} = \frac{941\mu C}{10\mu F} = \underline{\underline{94.1V}}$$

$$V_{C2} = \frac{Q_{C2}}{C2} = \frac{941\mu C}{20\mu F} = \underline{\underline{47.05V}}$$

$$V_{C3} = \frac{Q_{C3}}{C3} = \frac{941\mu C}{50\mu F} = \underline{\underline{18.82V}}$$

These three voltages will add up to the total voltage (160V), according to KVL.

7. Three capacitors valued at 10 μF, 20 μF, and 50μF are in parallel across a 100 V source. What is the charge on each capacitor? What is the total capacitance and charge?

a) In a parallel circuit, each capacitor has a charge related to its capacitance:

$$Q_{C1} = C_1V = (10\mu F)(100V) = 1mC$$
$$Q_{C2} = C_2V = (20\mu F)(100V) = 2mC$$
$$Q_{C3} = C_3V = (50\mu F)(100V) = 5mC$$

b) The total capacitance is the sum of the capacitors:

$$C_T = C1 + C2 + C3 = 10uF + 20uF + 50uF = \underline{\underline{80uF}}$$

The total charge is the sum of all the charges for parallel capacitors:

$$Q_T = Q1 + Q2 + Q3 = 1mC + 2mC + 5mC = \underline{\underline{8mC}}$$

9. Solve for the voltage V_R in a series R-C circuit after 1.2 time-constants if V_S is 200 V. What is the V_C value?

Assuming that the capacitor is initially discharged, the resistor voltage is expressed as:

$$v_R(t) = V_T e^{-t/\tau}$$

Since we want to know the answer after 1.2 τ, we substitute 1.2 for (t/τ) in the equation above and get:

$$v_R(t) = (200V)e^{-1.2} = \underline{\underline{60.24V}}$$

By KVL, the capacitor holds the remaining voltage:

$$V_C = V_T - V_R = 200V - 60.24V = \underline{\underline{139.76V}}$$

Remember that for an RC circuit, after 1 time-constant, the circuit is 63% charged, and that it takes approximately five (5) RC time-constants to get close to 100% charge. The problem requested an answer at slightly more than 1 τ (1.2 τ), so the capacitor voltage should be slightly more than 63% of 200 V, which would be 126 V.

(Chapter 13)

1. If the number of turns of a 100 mH inductor doubles, what happens to the total inductance? Nothing else is changed. Express in uH, mH, H.

   Since we know that inductance is found by: $L = \frac{\mu N^2 A}{l}$

   And *N* has been doubled, the inductance will increase by a factor of four (4X).

   This is the same as 400000 μH, 400 mH, or 0.4 H.

2. Current through an inductor is changing at 100 mA/s and the induced voltage is 30 mV. What is the inductance?

   $V = L\frac{\Delta I}{\Delta T}$ (Note: We will use $\frac{\Delta I}{\Delta T}$ to represent the rate of change instead of the calculus notation $\frac{di}{dt}$ utilized by the book. They represent the same idea.)

   $$L = \frac{V}{\frac{\Delta I}{\Delta T}} = \frac{30mV}{100mA/s} = 0.3H$$

3. What is the new inductance for a 100 μH air-core coil if its core is replaced with one having a relative permeability (μ_r) of 500?

   Since $L = \frac{\mu N^2 A}{l}$, we know that the inductance is directly proportional to the permeability μ. If μ is increased by a factor of 500, then the inductance also increases by the same factor (500X). The new inductance is therefore (500)(100 μH) = 50 mH.

4. How much energy is stored in the magnetic field of a 15 H inductor carrying 2A?

   $$W = \frac{1}{2}LI^2 = (1/2)(15H)(2A^2) = 30J$$

5. 100 mH, 0.2 H, and 1000 μH inductors are in series. Express the total inductance in mH assuming k_m=0.

   Series inductances add:

   $$L_T = L1 + L2 + L3 = 100mH + 200mH + 1mH = 301mH$$

6. What is the total inductance of a parallel combination of 10 H and 15 H devices? Assume that $k_m = 0$.

Parallel inductances work like parallel resistors as long as there is no magnetic coupling:

$$L_T = \frac{L1L2}{L1+L2} = \frac{(10H)(15H)}{(10H+15H)} = \underline{\underline{6H}}$$

7. A series RL circuit consists of a 250 mH inductor and a 100 Ω resistor. What is the time-constant? How long does it take to charge the inductor completely?

a) $\tau = L/R = 250mH/100\Omega = \underline{\underline{2.5ms}}$

b) It takes 5τ to charge to close to 100%. Therefore, 5τ is (5)(2.5 mS) = $\underline{\underline{\text{12.5mS}}}$