Homework 3 Solution Set

Blake Chapter 8 Problems 1-10
10 points (1 per problem)

1. Calculate the length (not specified in metric or English) of a practical half-wave dipole for a frequency of 150 MHz.

   A practical half-wave dipole will be approximately 95% of a half-wavelength:

$$\lambda = \frac{v}{f} = \frac{3\times 10^{8}\, m/s}{150 MHz} = 2.0m$$

$$L = (0.95)\frac{\lambda}{2} = (0.95)\frac{2m}{2} = \underline{\underline{0.95m}}$$

   Note: The text uses the formula $L = \frac{142.5}{f(MHz)}$ to find antenna length. You'll get the same answer with this method, but it's better to stick with basic formulas (like the wavelength equation above) and apply variations based on your understanding of the system. On most wire antennas, the 95% figure arises because the velocity of waves on the antenna is about 95% of the free-space value.

2. Calculate the efficiency of a dipole with a radiation resistance of 68 Ω and a total feedpoint resistance of 75 Ω.

   The efficiency of an antenna system can be found by:

$$\eta = \frac{P_{RAD}}{P_{IN}} = \frac{R_R}{R_{TOTAL}} = \frac{68\Omega}{75\Omega} = \underline{\underline{0.9066}} = \underline{\underline{90.66\%}}$$

3. Given that a half-wave dipole has a gain of 2.14 dBi, calculate the electric field strength at a distance of 10 km in free space in the direction of maximum radiation from a half-wave dipole that is fed, by means of lossless, matched line, by a 15 W transmitter.

   First, find the EIRP of the antenna:

$$EIRP = P_{IN}G_T = (15W)(10^{2.14/10}) = 24.55W$$

   (Note conversion of decibels back to a power ratio – the 10^(2.14/10) factor does this).

   Now use the electric field equation for free space:

$$E = \frac{\sqrt{30P_T}}{d} = \frac{\sqrt{(30)(24.55W)}}{10km} = \underline{\underline{2.7mV/m}}$$

4. Refer to the plot in Figure 8.38 and find the gain and beamwidth for the antenna shown.

   This figure is not well dimensioned, however, by inspection the gain appears to be approximately +5 dBi and the beamwidth (distance between –3 dB points) appears to be about 20°.

5. Calculate the EIRP in dBW for a 25 W transmitter operating into a dipole with 90% efficiency.

   The gain of a 100% efficient dipole is +2.15 dBi or 10^(2.15/10) [1.64] W/W. However, a lossy dipole has a gain of:

$$G_T = 1.64 \times \eta = 1.64 \times 0.90 = 1.47 W/W$$

   The radiated EIRP is therefore:

$$EIRP = P_T G_T = (25W)(1.47W/W) = 36.9W$$

   However, the answer was requested in dBW units:

$$dBW = 10\log\frac{P}{1W} = 10\log\frac{36.9W}{1W} = \underline{\underline{+15.67 dBW}}$$

6. Calculate the length (not specified in metric or English) of a quarter-wave monopole antenna for a frequency of 900 MHz.

   A quarter-wave antenna is just 95% of a quarter-wavelength (half of a half-wave dipole):

$$\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{900MHz} = 0.33\bar{3}m$$

$$L = (0.95)\frac{\lambda}{4} = (0.95)\frac{0.33\bar{3}m}{4} = \underline{\underline{0.079m}} = \underline{\underline{79mm}}$$

7. Calculate the optimum length (not specified in metric or English) of an automobile FM broadcast antenna, for operation at 100 MHz.

   The hidden assumption in this question is that the optimum antenna is a quarter-wave monopole. That may not necessarily be true, however, in terms of this assumption, the solution is the same as problem 6:

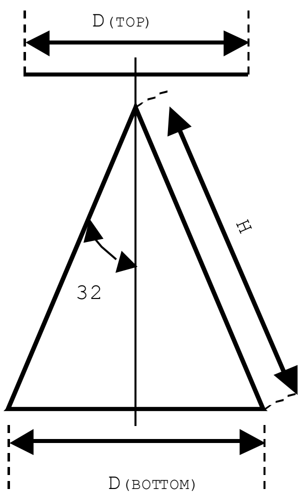
$$\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{100MHz} = 3m$$

$$L = (0.95)\frac{\lambda}{4} = (0.95)\frac{3m}{4} = \underline{\underline{0.7125m}} = \underline{\underline{71.25cm}}$$

8. Draw a dimensioned sketch of a discone antenna that will cover the VHF range from 30 to 300 MHz.

The following design parameters are derived from the *ARRL Antenna Book* (www.arrl.org):

H = hypotenuse; $H \approx \lambda/4$ at the lowest frequency of the unit.
$\Theta \approx 64°$ (total angle of the cone; each half is 32 °)
Disc diameter D_{TOP} should be about 70% of cone bottom diameter



From these parameters, we can derive the relationships by using some trigonometry:

$$\sin\Theta = \frac{opposite}{hypotenuse} = \frac{D_{BOTTOM}}{2H} \Big|^{\Theta=32}$$

$$0.53 = \frac{D_{BOTTOM}}{2H}$$

$$D_{BOTTOM} \approx 1.06H$$

$$D_{TOP} \approx 0.70 D_{BOTTOM} \approx (0.70)(1.06)H \approx 0.74H$$

To design the discone, let H be about a quarter-wavelength at the lowest frequency of interest, the apply the above relationships.

At 30 MHz, the dimensions in the figure will be:

$$H \approx \frac{\lambda}{4} \approx \left(\frac{3\times10^8 m/s}{30MHz}\right)\Big/4 \approx \underline{\underline{2.5m}}$$

$$D_{BOTTOM} \approx 1.06H \approx (1.06)(2.5m) \approx \underline{\underline{2.65m}}$$

$$D_{TOP} \approx 0.74H \approx (0.74)(2.5m) \approx \underline{\underline{1.85m}}$$

9. A helical antenna consists of 10 turns with a spacing of 10 cm and a diameter of 12.7 cm.

a) Calculate the frequency at which this antenna should operate.

The circumference of the turns is about one wavelength, and the spacing is 1/4 wavelength for this type of antenna. First, we must find the wavelength:

$$\lambda \approx \pi D \approx \pi(12.7cm) \approx 39.89cm$$

The spacing between the coils is 1/4 wavelength, and four times this value yields 40 cm. The frequency will be approximately:

$$f = \frac{3\times10^8 m/s}{\lambda} = \frac{3\times10^8 m/s}{39.89cm} = \underline{\underline{752MHz}}$$

(Note: Actual resonant frequency will probably slightly lower due to 95% velocity factor on conductor.)

b) Calculate the gain in dBi at the frequency found in part (a).

Equation 8.8 calculates the gain as a ratio:

$$G = \frac{15NS(\pi D)^2}{\lambda^3} = \frac{(15)(10turns)(10cm)(\pi\times12.7cm)^2}{(39.89cm)^3} = 37.6W/W$$

In dBi, this gain is expressed as:

$$G_{dBi} = 10\log(G) = 10\log(37.6W/W) = \underline{\underline{+15.75dBi}}$$

c) Calculate the beamwidth at the frequency found in part (a).

The beamwidth can be found by Equation 8.9:

$$\Theta_{\deg} = \frac{52\lambda}{\pi D}\sqrt{\frac{\lambda}{NS}} = \frac{52(39.89cm)}{\pi(12.7cm)}\sqrt{\frac{39.89cm}{(10T)(10cm)}} = \underline{\underline{32.83\deg}}$$

10. Assuming the aperture of a pyramidal horn is square, how large does it have to be to have a gain of 18 dBi at 12 GHz?

Equation 8.8 states that the gain of a rectangular horn antenna is:

$$G = \frac{7.5 d_E d_H}{\lambda^2}$$

Since our antenna's aperture is square, $d = d_E = d_H$ and we can write:

$$G = \frac{7.5 d^2}{\lambda^2}$$

Solving for d, the aperture cross section, we get:

$d = \lambda\sqrt{\frac{G}{7.5}}$ Where G = the required gain = $10^{18dbi/10} = 63W/W$ and $\lambda = \frac{3\times10^8 m/s}{12GHz} = 25mm$

$$d = \lambda\sqrt{\frac{G}{7.5}} = 25mm\sqrt{\frac{63W/W}{7.5}} = \underline{\underline{72.4mm}}$$