Homework 5 Solution Set

Blake Chapter 7 Problems 1-11
11 points (1 per problem)

1. Find the propagation velocity of radio waves in glass with a relative permittivity of 7.8

$$v_p = c \times VF = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8 m/s}{\sqrt{7.8}} = \underline{\underline{1.074\times10^8 m/s}}$$

2. Find the wavelength in free space for radio waves at each of the following frequencies:

a) 50 kHz: $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{50kHz} = \underline{\underline{6km}}$

b) 1 MHz: $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{1MHz} = \underline{\underline{300m}}$

c) 23 MHz: $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{23MHz} = \underline{\underline{13.04m}}$

d) 300 MHz: $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{300MHz} = \underline{\underline{1m}}$

e) 450 MHz: $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{450MHz} = \underline{\underline{66.\bar{6}cm}}$

f) 12 GHz: $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{12GHz} = \underline{\underline{25mm}}$

3. An isotropic source radiates 100 W of power in free space. At a distance of 15 km from the source, calculate the power density and the electric field intensity.

a) $\mathscr{P} = \frac{P_t G_t}{4\pi d^2} = \frac{(100W)(1)}{4\pi(15km)^2} = \underline{\underline{35.4nW/m^2}}$

b) $\mathscr{E} \approx \frac{\sqrt{30 P_t G_t}}{d} \approx \frac{\sqrt{(30)(100W)(1)}}{15km} \approx \underline{\underline{3.65mV/m}}$

4. A certain antenna has a gain of 7 dBi.

a) What is its effective area if it operates at 200 MHz?

$$A_{eff} = \frac{\lambda^2 G_R}{4\pi}$$ where G_R is $10^{(dBi/10)} = 10^{(7/10)}$=5.01W/W, and $\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{200MHz} = \underline{\underline{1.5m}}$

$$A_{eff} = \frac{\lambda^2 G_R}{4\pi} = \frac{(1.5m)^2(5.01W/W)}{4\pi} = \underline{\underline{0.897m^2}}$$

b) How much power would it absorb from a signal with a field strength of 50 μV/m?

$\mathscr{P} = \mathscr{E}^2 / \mathscr{Z}$ = (50 μV/m)² / 377 Ω = 6.63 pW/m²

$$P_R = A \times P = (0.897m^2)(6.63pW/m^2) = \underline{\underline{5.95pW}}$$

5. Find the characteristic impedance of glass with a relative permittivity of 7.8 (implied: relative permeability of 1.0).

$$Z = \frac{Z_0}{\sqrt{\varepsilon_r}} = \frac{377\Omega}{\sqrt{7.8}} = \underline{\underline{135\Omega}}$$

Note: We can also use the following basic relationship to find the characteristic impedance:

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{(1.26\times10^{-6}H/m)(1.0)}{(8.85\times10^{-12}F/m)(7.8)}} = \underline{\underline{135\Omega}}$$

6. A transmitter has an output power of 50 W. It is connected to its antenna by a feedline that is 25 m long and properly matched. The loss in the feedline is 5 dB/100 m. The antenna has a gain of 8.5 dBi.

a) How much power reaches the antenna?

The feedline loss is (25 m x 5 dB/ 100m) or 1.25 dB. The power delivered is:

$$P_{out} = P_{in} \times 10^{(dB/10)} = 50W(10^{(-1.25/10)}) = \underline{\underline{37.5W}}$$

b) What is the EIRP in the direction of maximum antenna gain?

$$EIRP = P_{ant} \times G_t = 37.5W \times 10^{(8.5dBi/10)} = \underline{\underline{265.4W}}$$

c) What is the power density 1 km from the antenna in the direction of maximum gain, assuming free space propagation?

$$\mathscr{P} = \frac{P_t G_t}{4\pi d^2} = \frac{(264.4W)}{4\pi(1km)^2} = \underline{\underline{21.12\mu W/m^2}}$$

d) What is the electric field strength at the same place as in (c)?

$$\mathscr{E} \approx \frac{\sqrt{30 P_t G_t}}{d} \approx \frac{\sqrt{(30)(265.4W)}}{1km} \approx \underline{\underline{89.2mV/m}}$$

7. A satellite transmitter operates at 4 GHz with an antenna gain of 40 dBi. The receiver, 40,000 km away, has an antenna gain of 50 dBi. If the transmitter has a power of 8 W (ignoring feedline losses and mismatch), find:

a) The EIRP in dBw

Transmitted power: $dBW = 10\log\frac{P}{1W} = 10\log\frac{8W}{1W} = +9.03dBW$

$$EIRP_{dBW} = P_{t(dBW)} + G_{t(dBi)} = 9.03dBW + 40dBi = \underline{\underline{+49.03dBW}}$$

b) The power delivered to the receiver

$$FSPL = 32.44 + 20\log f(MHz) + 20\log d(km) = 32.44 + 20\log 4000 + 20\log 40{,}000 = 196.5dB$$

$$P_R = EIRP - FSPL + G_{r(dBi)} = 49.03dBW - 196.5dB + 50dBi = \underline{\underline{-97.5dBW}} = \underline{\underline{-67.5dBm}}$$

$$P_R(watts) = 1watt \times 10^{(-97.5dBW/10)} = \underline{\underline{178pW}}$$

8. A paging system has a transmitting antenna located 50 m above average terrain (HAAT). How far away could the signal be received by a pager carried 1.2 m above the ground?

$$d_{km} = \sqrt{17h_t} + \sqrt{17h_r} = \sqrt{17(50m)} + \sqrt{17(1.2)} = \underline{\underline{33.7km}}$$

9. A boat is equipped with a VHF marine radio, which it uses to communicate with other nearby boats and shore stations. If the antenna on the boat is 2.3 m above the water, calculate the maximum distance for communication with:

a) another similar boat: $d_{km} = \sqrt{17h_t} + \sqrt{17h_r} = \sqrt{17(2.3m)} + \sqrt{17(2.3)} = \underline{\underline{12.5km}}$

b) a shore station with an antenna on a tower 22 m above the water level

$$d_{km} = \sqrt{17h_t} + \sqrt{17h_r} = \sqrt{17(2.3m)} + \sqrt{17(22)} = \underline{\underline{25.6km}}$$

c) another boat, but using the shore station as a repeater (assuming that the repeater can be in the middle); the distance will be just 2X the distance to the repeater.

$$d_{km} = d_1 + d_2 = 25.6km + 25.6km = \underline{\underline{51.2km}}$$

10. A PCS signal at 1.9 GHz arrives at an antenna via two paths differing in length by 19 m.

a) Calculate the difference in arrival time for the two paths.

$$T = \frac{D}{R} = \frac{19m}{3\times10^8 m/s} = \underline{\underline{63.3ns}}$$

b) Calculate the phase difference between the two signals.

$$\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{1.9GHz} = \underline{\underline{0.1579m}}$$

$\Phi = \frac{D}{\lambda}\times 360 = \frac{19m}{0.1579m}\times 360 = 43{,}320^o$ (This is many wavelengths, so we use the fractional portion by dividing by 360 degrees.) By using the *fraction* of 360 in the above result, we get

$\Phi = 360\times\left(\frac{43320}{360} - \left\lfloor \frac{43320}{360} \right\rfloor\right) = \underline{\underline{120^o}}$ where $\lfloor\ \rfloor$ denotes the next-smallest integer (`floor()`) function.

11. Use the mobile-propagation model given in Equation (7.18) to calculate the loss over a path of 5 km, with a base antenna 25 m above the ground, for a

a) cellular telephone at 800 MHz

$$L_p = 68.75 + 26.16\log f(MHz) - 13.82\log h(meters) + (44.9 - 6.55\log h(meters))\log d(km)$$

$$L_p = 68.75 + 26.16\log(800) - 13.82\log(25) + (44.9 - 6.55\log(25)\log(5) = \underline{\underline{150.35dB}}$$

b) PCS at 1900 MHz

$$L_p = 68.75 + 26.16\log(1900) - 13.82\log(25) + (44.9 - 6.55\log(25)\log(5) = \underline{\underline{160.2dB}}$$

Note that this model doesn't use the height of the receiver's antenna. It assumes that the receiver is within 1 to 2 meters above the ground.