ECT150
Homework #9 Problem Set
Sr. Professor Wheeler

Chapter 14 problems 1-7
Chapter 18 problems 1-3,5,6,8,9

Total Points: 28 (2 per problem)

All work must be shown, and final answers boxed or underlined. No credit if work is not shown.

(Chapter 14)

1. What is the inductance of an inductor that has 376.8 Ohms reactance at a frequency of 100 Hz? What is X_L at 450 Hz?

$$X_L = \omega L = 2\pi f L$$
$$\therefore L = \frac{X_L}{2\pi f} = \frac{376.8\Omega}{2\pi(100Hz)} = \underline{\underline{599.7mH}}$$

At 450 Hz:

$$X_L = \omega L = 2\pi f L = 2\pi(450Hz)(599.6mH) = \underline{\underline{1695.6\Omega}}$$

Note that since the frequency increased by a factor of 4.5, X_L also increased by the same factor.

2. What is the X_L of a 5H inductor at 120 Hz? At 600 Hz?

$$X_L = \omega L = 2\pi f L = 2\pi(120Hz)(5H) = \underline{\underline{3770\Omega}}$$

$$X_L = \omega L = 2\pi f L = 2\pi(600Hz)(5H) = \underline{\underline{18.85k\Omega}}$$

3. What is the total inductance of Figure 14-15? When the voltage applied is 100 V and the current through L1 is 100 mA, what is the frequency of the applied voltage?

Figure 14-15 shows 4 mH, 4 mH, and 6 mH inductors in parallel. Assuming $k_m = 0$ (no magnetic coupling between the devices), we get:

$$L_T = \frac{1}{1/L1 + 1/L2 + 1/L3} = \frac{1}{1/4mH + 1/4mH + 1/6mH} = \underline{\underline{1.5mH}}$$

L1 is a 4 mH inductor with 100 mA flowing through it with 100V applied. We can write:

$$I = \frac{V}{X_{L1}}$$

$$\therefore X_{L1} = \frac{V}{I} = \frac{100V}{100mA} = 1k\Omega$$

Since we know X_L and L, we can get frequency:

$$X_L = \omega L = 2\pi f L$$

$$\therefore f = \frac{X_L}{2\pi L} = \frac{1k\Omega}{2\pi(4mH)} = \underline{\underline{39.79kHz}}$$

4. Find L_T for the circuit in Figure 14-16. At approximately what frequency does the total reactance equal 5k Ohms?

Figure 14-16 shows inductors valued 1H, 2H, 3H, 4H, and 5H in series. Assuming k_m to be zero (no mutual coupling):

$$L_T = L1 + L2 + L3 + L4 + L5 = 1H + 2H + 3H + 4H + 5H = \underline{\underline{15H}}$$ (BIG inductance!)

Solving for frequency (given inductance and reactance) we get:

$$X_L = \omega L = 2\pi f L$$

$$\therefore f = \frac{X_L}{2\pi L} = \frac{5k\Omega}{2\pi(15H)} = \underline{\underline{53Hz}}$$

5. Find the Q of the coil shown in Figure 14-17. When the frequency triples and R of the inductor doubles, does Q increase, decrease, or remain the same? If the applied voltage were DC instead of AC, would Q change? Explain.

For any series reactive circuit, Q is expressed by:

$$Q_S = \frac{X_S}{R_S} = \frac{2\pi f L_S}{R_S} = \frac{2\pi(2MHz)(10\mu H)}{10\Omega} = \underline{\underline{12.6}}$$

If f triples but R_S only doubles, Q still increases since X increases faster than R IN THIS CASE.

If DC is applied, Q_S becomes zero because X for an inductor is zero at DC.

6. Draw a waveform showing two cycles that illustrates the circuit V and I for Figure 14-18. Appropriately label the peak values of V and I.

The current I is:

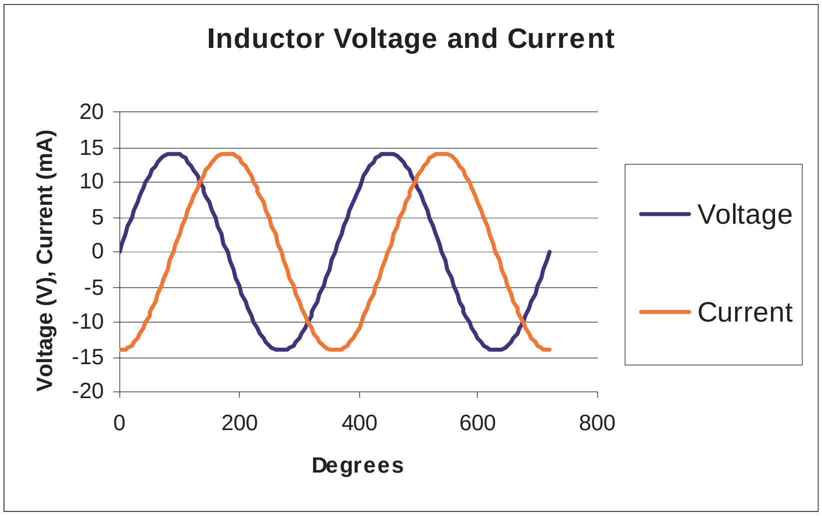
$I = \frac{V}{X} = \frac{10V}{1k} = 10mA\angle -90°$ (This is an RMS current since V is an RMS voltage, and inductor voltage leads inductor current by 90 degrees; "ELI").

The peak voltages and currents are:

$$V_{pk} = V\sqrt{2} = 10\sqrt{2} = \underline{14.1Vpk}$$
$$I_{pk} = I\sqrt{2} = 10mA\sqrt{2} = \underline{14.1mApk}$$

Knowing these, we can construct the graph:

The graphs were created using a Excel. The filename is "ELI_PLOT.XLS".



The voltage leads the current by 90 degrees in an inductor.

7. Find V_T, V_{L1}, and L_T for Figure 14-19.

In Figure 14-19, the current is first calculated:

$$I = \frac{V_{L2}}{X_{L2}}$$

But X_{L2} isn't known (we don't know the frequency). However, we do know that the reactance of L1 is 20k (and its inductance is 10H). Therefore, the frequency is:

$$X_L = \omega L = 2\pi f L$$

$$\therefore f = \frac{X_{L1}}{2\pi L1} = \frac{20k\Omega}{2\pi(10H)} = 318.3Hz$$

With this information, we can now find X_{L2}:

$$X_{L2} = \omega L_2 = 2\pi f L_2 = 2\pi(318.3Hz)(20H) = 40k\Omega$$

Now we can find the current in L_2:

$$I = \frac{V_{L2}}{X_{L2}} = \frac{30V}{40k\Omega} = 0.75mA$$

The total voltage in the circuit is:

$$V_T = I_T X_T = 0.75mA(20k + 40k) = \underline{\underline{45V}}$$

The voltage across L1 is:

$$V_{L1} = X_{L1} I_{L1} = (20k)(0.75mA) = \underline{\underline{15V}}$$ (Could have also used KVL)

The total inductance is the sum of L1 and L2:

$$L_T = L1 + L2 = 10H + 20H = \underline{\underline{30H}}$$

(Chapter 18 problems)

1. What is the reactance of a 0.00025 μF capacitor operating at 5 kHz?

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(5kHz)(0.00025\mu F)} = 127.3k\Omega$$

2. What capacitance value has 250 Ohms reactance at 1500 Hz?

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(1500Hz)(250\Omega)} = \underline{\underline{0.42\mu F}}$$

3. The frequency of applied voltage to a given circuit triples while the capacitance remains the same. What happens to the capacitive reactance?

Since $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ we know that X_C is inversely proportional to frequency. If frequency increases by 3X, then X_C becomes 1/3 of its original value.

5. For Figure 18-12 find C_T, X_{CT}, V_A, C_1, C_2, and X_{C2}.

$$X_{CT} = \frac{V_T}{I_T} = \frac{(20V + 50V)}{2mA} = \underline{\underline{35k\Omega}}$$

$$C_T = \frac{1}{2\pi f X_{CT}} = \frac{1}{2\pi(10Hz)(35k\Omega)} = \underline{\underline{0.46\mu F}}$$

$V_A = V_T = I_T X_T = (2mA)(35k\Omega) = \underline{\underline{70V}}$ (Same as total drop across C1 and C2, above)

To find C1 we need to know X_{C1}: $X_{C1} = \frac{V_{C1}}{I_{C1}} = \frac{50V}{2mA} = 25k\Omega$

Then we can solve for C1:

$$C_1 = \frac{1}{2\pi f X_{C1}} = \frac{1}{2\pi(10Hz)(25k\Omega)} = \underline{\underline{0.64\mu F}}$$

$$X_{C2} = \frac{V_{C2}}{I_{C2}} = \frac{20V}{2mA} = \underline{\underline{10k\Omega}}$$

$$C_2 = \frac{1}{2\pi f X_{C2}} = \frac{1}{2\pi(10Hz)(10k\Omega)} = \underline{\underline{1.59\mu F}}$$

6. For Figure 18-12, if C1 and C2 were equal in value, what would be the new voltages on C1 and C2?

Equal capacitors would drop equal voltages in series. Two capacitors will divide the total voltage in half. By inspection, $V_{C1} = V_{C2} = V_T / 2 = 35$ V under this condition.

8. For Figure 18-13 find C_T, I_1, I_2, X_{CT}, X_{C1}, X_{C2}, and I_T.

Figure 18-13 has a 20 Hz 25V source driving C1, 2 µF and C2, 1 µF in parallel.

$C_T = C1 + C2 = 2\mu F + 1\mu F = \underline{\underline{3uF}}$ (Parallel capacitances add)

$$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(20Hz)(2\mu F)} = \underline{\underline{3979\Omega}}$$

$$X_{C2} = \frac{1}{\omega C_2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(20Hz)(1\mu F)} = \underline{\underline{7958\Omega}}$$

$$X_{CT} = \frac{1}{\omega C_T} = \frac{1}{2\pi f C_T} = \frac{1}{2\pi(20Hz)(3\mu F)} = \underline{\underline{2653\Omega}}$$ (Could also do as X1 || X2)

$$I_1 = \frac{V_{C1}}{X_{C1}} = \frac{25V}{3979\Omega} = \underline{\underline{6.28mA}}$$

$$I_1 = \frac{V_{C2}}{X_{C2}} = \frac{25V}{7958\Omega} = \underline{\underline{3.14mA}}$$

$$I_T = I_1 + I_2 = 6.28mA + 3.14mA = \underline{\underline{9.42mA}}$$

9. In Figure 18-13, if C2 is equal to C1 (both 2 µF), what is I_T?

The total current will become twice the current in C1 (solved in problem #8):

$$I_T = 2I_1 = 2(6.28mA) = \underline{\underline{12.57mA}}$$