

Homework 1 Solution Set

Blake Chapter 6 Problems 1-8

8 points (1 per problem)

1. Find the wavelength for radio waves in free space at each of the following frequencies:

$$\text{a) } 160 \text{ MHz: } \lambda = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/s}}{160 \text{ MHz}} = \underline{\underline{1.875 \text{ m}}}$$

$$\text{b) } 800 \text{ MHz: } \lambda = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/s}}{800 \text{ MHz}} = \underline{\underline{0.375 \text{ m}}}$$

$$\text{c) } 2 \text{ GHz: } \lambda = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/s}}{2 \text{ GHz}} = \underline{\underline{0.15 \text{ m}}}$$

2. How far does a radio wave travel through space in one microsecond?

$$D = RT = (3 \times 10^8 \text{ m/s})(1 \mu\text{s}) = \underline{\underline{300 \text{ m}}}$$

3. Visible light has a range of wavelengths from approximately 400 nanometers (violet) to 700 nm (red). Express this as a frequency range.

Since $\lambda = \frac{v}{f}$, we can rearrange the equation to solve for frequency:

$$f_{\text{RED}} = \frac{v}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{700 \text{ nm}} = \underline{\underline{4.2857 \times 10^{14} \text{ Hz}}} = \underline{\underline{428.57 \text{ THz}}}$$

$$f_{\text{VIOLET}} = \frac{v}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{400 \text{ nm}} = \underline{\underline{7.5 \times 10^{14} \text{ Hz}}} = \underline{\underline{750 \text{ THz}}}$$

4. Calculate the characteristic impedance of an open wire transmission line consisting of two wires with diameter 1 mm and separation 1 cm.

$$Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln\left(\frac{2s}{d}\right) = \frac{120}{\sqrt{1}} \ln\left(\frac{2(1 \text{ cm})}{1 \text{ mm}}\right) = \underline{\underline{359 \Omega}}$$

Note: ϵ_r is the dielectric constant of the insulator, air in this case. Textbook uses the formula $Z_0 = 276 \log(D/r)$ which gives same result, but only for air dielectric lines.

5. Calculate the characteristic impedance of a coaxial line with a polyethylene dielectric, if the diameter of the inner conductor is 3 mm and inside diameter of the outer conductor is 10 mm.

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{D}{d}\right) = \frac{60}{\sqrt{2.3}} \ln\left(\frac{10\text{mm}}{3\text{mm}}\right) = \underline{\underline{47.6\Omega}}$$

Note: Polyethylene has a dielectric constant of 2.3. Textbook uses the formula $Z_0 = 138/\sqrt{\epsilon_r} * \log(D/d)$ which is equivalent.

6. Repeat problem 1 for waves on a coaxial cable with a solid polyethylene dielectric.

To solve these, find the velocity of signals on the cable. First, calculate the velocity factor:

$$VF = \frac{1}{\sqrt{\epsilon_r}} = \frac{1}{\sqrt{2.3}} = 0.6593 \text{ (You need to know the dielectric type; given as polyethylene)}$$

The velocity of propagation is:

$$v = c \times VF = (3 \times 10^8 \text{ m/s})(0.6593) = 1.978 \times 10^8 \text{ m/s}$$

With this information in hand, we can now find the wavelengths:

$$\text{a) } 160 \text{ MHz: } \lambda = \frac{v}{f} = \frac{1.978 \times 10^8 \text{ m/s}}{160 \text{ MHz}} = \underline{\underline{1.236 \text{ m}}}$$

$$\text{b) } 800 \text{ MHz: } \lambda = \frac{v}{f} = \frac{1.978 \times 10^8 \text{ m/s}}{800 \text{ MHz}} = \underline{\underline{0.247 \text{ m}}}$$

$$\text{c) } 2 \text{ GHz: } \lambda = \frac{v}{f} = \frac{1.978 \times 10^8 \text{ m/s}}{2 \text{ GHz}} = \underline{\underline{0.099 \text{ m}}}$$

7. Repeat problem 2 for a radio wave propagating along a coaxial cable with polyethylene foam dielectric.

First, calculate the new velocity of propagation. NOTE that the dielectric constant for polyfoam varies with its composition (it is polyethylene blown up with nitrogen gas, which is inert.)

$$VF = \frac{1}{\sqrt{\epsilon_r}} = \frac{1}{\sqrt{1.3}} = 0.877$$

$$v = c \times VF = (3 \times 10^8 \text{ m/s})(0.877) = 2.63 \times 10^8 \text{ m/s}$$

$$D = RT = (2.63 \times 10^8 \text{ m/s})(1 \mu\text{s}) = \underline{\underline{263 \text{ m}}}$$

8. How long a line is required to produce a 45 degree phase shift at 400 MHz if the dielectric is

a) air?

First, find the wavelength (which is 360 degrees of electrical length):

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/s}}{400 \text{ MHz}} = 0.75 \text{ m} \quad (\text{Since dielectric is air, } v=3 \times 10^8 \text{ m/s, same as free space})$$

Now translate that into a physical length, knowing that 360 degrees = 0.75 m:

$$L = \lambda \frac{\phi}{360} = (0.75 \text{ m}) \frac{45}{360} = \underline{\underline{0.09375 \text{ m}}}$$

b) solid polyethylene?

The velocity of propagation will fall, shortening the wavelength:

$$VF = \frac{1}{\sqrt{\epsilon_r}} = \frac{1}{\sqrt{2.3}} = 0.6593$$

$$v = c \times VF = (3 \times 10^8 \text{ m/s})(0.6593) = 1.978 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{1.978 \times 10^8 \text{ m/s}}{400 \text{ MHz}} = 0.494 \text{ m}$$

Finally, the physical length will now be:

$$L = \lambda \frac{\phi}{360} = (0.494 \text{ m}) \frac{45}{360} = \underline{\underline{0.062 \text{ m}}}$$