Doppler RF Direction Finding: Algorithms

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The basis for possible software algorithms for a Doppler-type radio direction finder is relatively straightforward. It relies on the geometric model of antenna switching unit action. The physical model is shown in Figure 1.

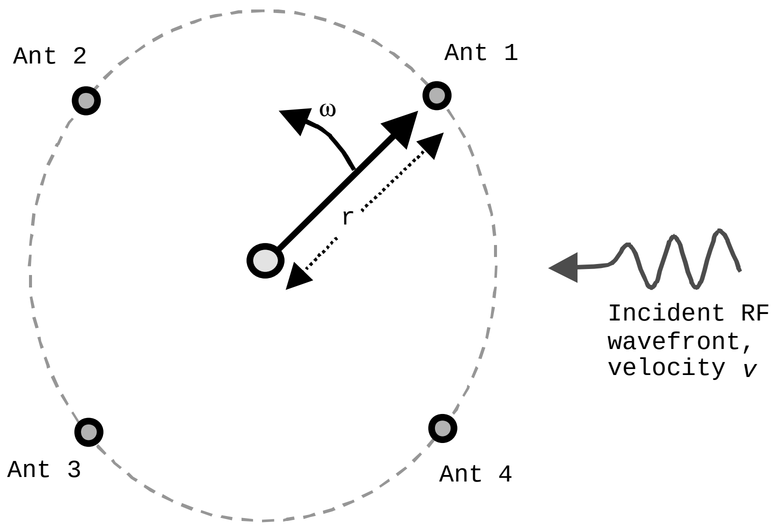

Figure 1: Physical Model of Doppler DF System

Since we know that $s = r\theta$ and $\frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$ (the tangential velocity of the rotating antenna assembly), we can resolve the perpendicular (x) component Vq of the Doppler velocity differential by writing:

(1) $Vq(t) = r\omega\cos\left(\theta(t)\right) = r\omega\cos(\omega t + \phi_s)$

Where ϕ_s is the bearing to the RF energy source, r is the radius of the rotating antenna assembly, and ω is the frequency of that rotation.

The instantaneous Doppler shift of the incoming RF carrier can be described as:

(2) $\Delta f(t) = \frac{Vq(t)}{\lambda} = \frac{r\omega\cos(\omega t + \phi_s)}{\lambda}$

Where $\lambda = \frac{c}{f_c}$, c being the speed of the incident wave, and f_c being its carrier frequency.

From equation (2) we can readily determine that the peak frequency change δ is:

(3) $$\delta = \frac{r\omega}{\lambda} \text{ (r/s)}$$

Or in more convenient terms,

(4) $$\delta = \frac{r\omega}{2\pi\lambda} \text{ (Hz)}$$

Since we know that the effective Doppler modulation frequency is ω, we can calculate the FM modulation index (and with that, the optimum receiver spectral mask) as follows:

(5) $$m_f = \frac{\delta}{f_m} = \frac{\left(\frac{r\omega}{2\pi\lambda}\right)}{\left(\frac{\omega}{2\pi}\right)} = \frac{r}{\lambda}$$

Equation 5 tells us that the modulation index is frequency dependent - as the carrier frequency is increased, the modulation index also increases in a like manner. It also shows that the larger the radius of rotation *r*, the more effective phase shaft that is imparted to the RF carrier wave.

Of course, if *r* is increased beyond approximately 0.35 λ (the distance between adjacent antennas must be less than 0.5 λ), EM field aliasing will occur (this is a sampled data system due to discrete antenna switching) and the phase results will become ambiguous. For a given antenna configuration, this limits the maximum useful frequency of a unit. This also limits the maximum possible phase modulation index to 0.35 in any particular case, since the maximum ratio of *r* to λ (Equation 5) is 0.35 in a practical design.

In a Doppler DF using a rotational radius *r* of about 12" (0.3 m), an incoming carrier frequency of 146 MHz (wavelength of 2.05 m), and an angular frequency of 800 Hz (5026.55 r/s), the expected carrier frequency deviation will be 117 Hz, and the modulation index will be 0.15 rad. This is a very narrow FM signal. The Bessel coefficients $J_n(0.15)$ for this signal are shown below.

n	Jn(0.15)	Jn(0.15), dBc
0	0.9944	-0.48
1	0.0748	-22.5
2	0.0028	-51.1
3	0.0000702	-83.1

From this information we can see that this configuration only uses about 1600 Hz of the receiver's available bandwidth of approximately 10 KHz. Increasing the rotational frequency might be desirable in that it would increase the signal bandwidth to more fully fill the receiver IF.

A receiver dedicated to this task should have a very narrow IF filter to fully utilize the available information power in the recovered signal. Conventional amateur and commercial FM receivers are therefore not optimal for this application.

Response of the FM Receiver

After recovery and detection of the RF signal, it will be fed into an FM discriminator. The signal at the output of an FM detector can be characterized as follows:

$$(6)\ Vo = \gamma \frac{r\omega}{\lambda} \cos(\omega t + \theta_{static} + \phi_s)$$

Where γ is the FM detector conversion gain in V/Hz,
r is the radius of the rotating antenna,
ϖ is the frequency of antenna rotation,
λ is the wavelength of the incoming carrier,
ϕ_s is the bearing to the RF source, and
θ_{static} is the static error caused by physical receiver and transmitter characteristics.

The receiver's primary function in determining the bearing to the RF source is to act as a coherent phase detector. The error term θ_{static} can be eliminated by subtracting it from a calibration constant stored by software.

Figure 2 shows one half of a proposed detection scheme.

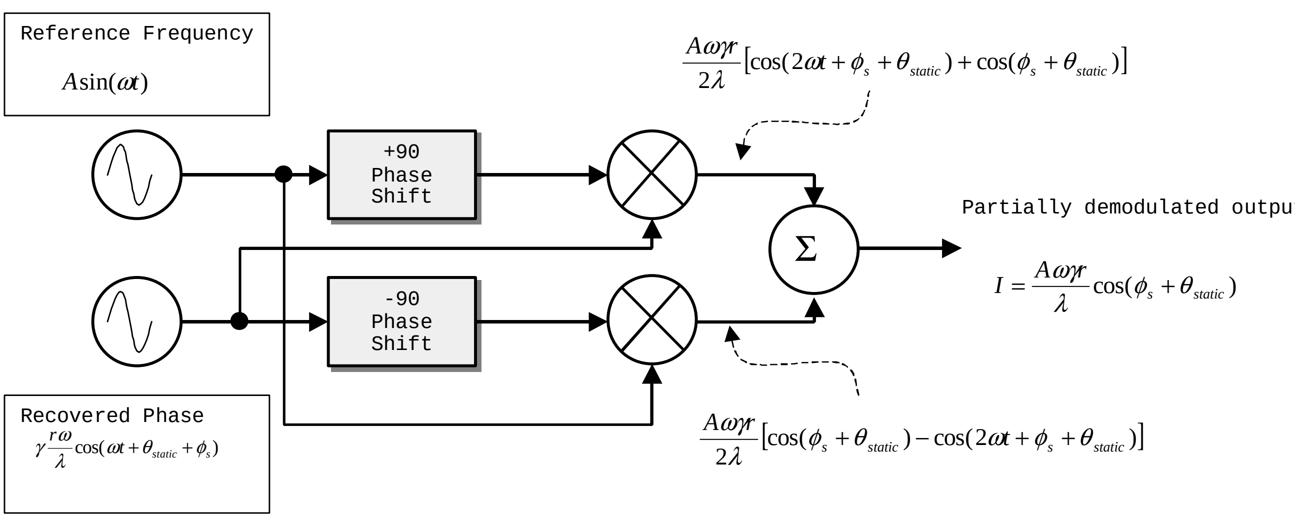

Figure 2: Proposed Detection Scheme

In Figure 2, an in-phase term is developed by the circuit as follows:

$$(7)\quad I = \frac{A\omega\gamma r}{\lambda}\cos(\phi_s + \theta_{static})$$

Likewise (but not shown), an out-of-phase (quadrature) term can be developed:

$$(8)\quad Q = \frac{A\omega\gamma r}{\lambda}\sin(\phi_s + \theta_{static})$$

By combining both of these terms, the phase can be determined:

$$(9)\quad \frac{I}{Q} = \tan(\phi_s + \theta_{static})$$

Such that:

$$(10)\quad \phi_s + \theta_{static} = \tan^{-1}\left(\frac{I}{Q}\right) \text{ for non-zero Q.}$$

The quadrant of the angle can be determined by individually examining the signs of I and Q. When Q is zero, the angle is either +90 (I positive) or -90 (I negative).

Proposed DSP Architecture

The following are points to be considered for one possible DSP implementation of the above scheme.

- ω = 6283 r/s (1 KHz antenna rotation frequency, approximate)
- f_s = 32 KHz (32 X the fundamental). The sine wave lookup table will need 32 points. A shift of 8 sample positions is 90 degrees. 32 points is 360 degrees.
- The antenna unit is clocked every 8 samples (90 degrees).
- A multi-pole FIR synchronously-tuned bandpass would be placed at the DSP input.